SpeedLabs

MATHS

CBSE 9th

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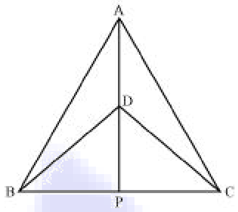
Q.1 Δ ABC and Δ DBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (See figure). If AD is extended to intersect BC at P, show that:



(i) Δ ABD ≅ Δ ACD

(ii) Δ ABP ≅ Δ ACP

(iii) AP bisects ∠ A as well as ∠ D.

(iv) AP is the perpendicular bisector of BC.

Ans. **(i)** Δ ABC is an isosceles triangle.

∴ AB = AC

Δ DBC is an isosceles triangle.

∴ BD = CD

Now in Δ ABD and Δ ACD,

AB = AC [Given]

BD = CD [Given]

AD = AD [Common]

∴ Δ ABD ≅ Δ ACD [By SSS congruency]

⇒ ∠ BAD = ∠ CAD [By C.P.C.T.] ....(i)

(ii) Now in Δ ABP and Δ ACP,

AB = AC [Given]

∠ BAD = ∠ CAD [From eq. (i)]

AP = AP

∴ Δ ABP ≅ Δ ACP [By SAS congruency]

(iii) Since ABP ACP [From part (ii)]

⇒ ∠ BAP = ∠ CAP [By C.P.C.T.]

⇒ AP bisects ∠ A.

Since Δ ABD ≅ Δ ACD [From part (i)]

⇒ ∠ ADB = ∠ ADC [By C.P.C.T.] ....(ii)

Now $\angle$ ADB + $\angle$ BDP = [Linear pair] ……….(iii)

And $\angle$ ADC + $\angle$ CDP = [Linear pair] ………. (iv)

From eq. (iii) and (iv),

$\angle$ ADB + $\angle$ BDP = $\angle$ ADC + $\angle$ CDP

$\Rightarrow$ $\angle$ ADB + $\angle$ BDP = $\angle$ ADB + $\angle$ CDP [Using (ii)]

$\Rightarrow$ $\angle$ BDP = $\angle$ CDP

$\Rightarrow$ DP bisects D or AP bisects D.

(iv) Since Δ ABP $\cong$ Δ ACP [From part (ii)]

$\therefore$ BP = PC [By C.P.C.T.] ………. (v)

And $\angle$ APB = $\angle$ APC [By C.P.C.T.] ……. (vi)

Now $\angle$ APB + $\angle$ APC = 180° [Linear pair]

$\Rightarrow$ $\angle$ APB + $\angle$ APC = 180° [Using eq. (vi)]

$\Rightarrow$ 2 $\angle$ APB = 180°

$\Rightarrow$ APB = 90°

$\Rightarrow$ AP $\perp$ BC ………. (vii)

From eq. (v), we have BP PC and from (vii), we have proved AP $\perp$ B. So, collectively AP is perpendicular bisector of BC.

Q.2 AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that:

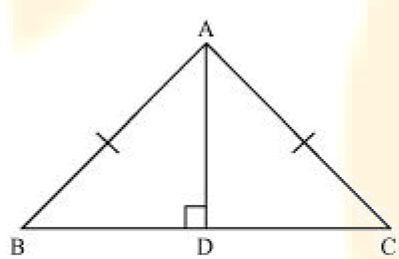
(i) AD bisects BC.

(ii) AD bisects $\angle$ A.

Ans. In Δ ABD and Δ ACD,

AB = AC [Given]

$\angle$ ADB = $\angle$ ADC = 90° [AD $\perp$ BC]



AD = AD [Common]

$\therefore$ Δ ABD $\cong$ ACD [RHS rule of congruency]

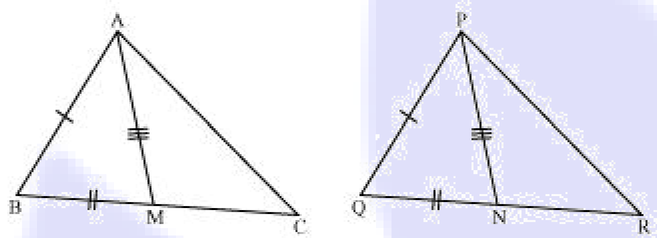
$\Rightarrow$ BD = DC [By C.P.C.T.]

⇒ AD bisects BC

Also ∠ BAD = ∠ CAD [By C.P.C.T.]

⇒ AD bisects ∠ A.

Q.3 Two sides AB and BC and median AM of the triangle ABC are respectively equal to side PQ and QR and median PN of Δ PQR (See figure). Show that:



(i) Δ ABM ≅ Δ PQN

(ii) Δ ABC ≅ Δ PQR

Ans. AM is the median of Δ ABC.

∴ BM = MC = $\frac{1}{2}$ BC .... (i)

PN is the median of PQR.

∴ QN = NR = $\frac{1}{2}$ QR .... (ii)

Now BC = QR [Given] $\frac{1}{2}$ BC = QR

∴ BM = QN .... (iii)

(i) Now in Δ ABM and Δ PQN,

AB = PQ [Given]

AM = PN [Given]

BM = QN [From eq. (iii)]

∴ Δ ABM ≅ Δ PQN [By SSS congruency]

⇒ ∠ B = ∠ Q [By C.P.C.T.] ....(iv)

(ii) In Δ ABC and Δ PQR,

AB = PQ [Given]

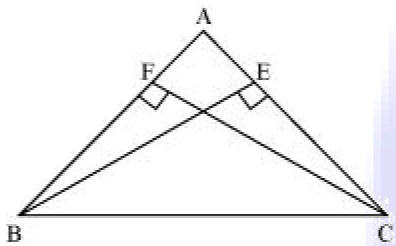
∠ B = ∠ Q [Prove above]

BC = QR [Given]

∴ Δ ABC ≅ Δ PQR [By SAS congruency]

Q.4 BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Ans. In Δ BEC and Δ CFB,



∠ BEC = ∠ CFB [Each 90°]

BC = BC [Common]

BE = CF [Given]

∴ Δ BEC ≅ ΔCFB [RHS congruency]

⇒ EC = FB [By C.P.C.T.] …. (i)

Now In Δ AEB and Δ AFC

∠ AEB = ∠ AFC [Each 90°]

∠ A = ∠ A [Common]

BE = CF [Given]

∴ Δ AEB ≅ Δ AFC [ASA congruency]

⇒ AE = AF [By C.P.C.T.] …………(ii)

Adding eq. (i) and (ii), we get,

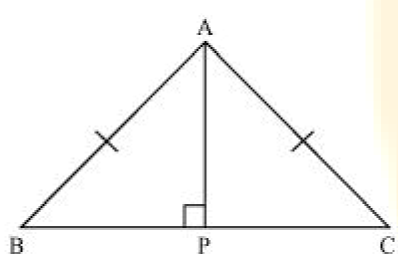
EC + AE = FB + AF

⇒ AB = AC

⇒ ABC is an isosceles triangle.

Q.5 ABC is an isosceles triangle with AB = AC. Draw AP ∥ BC and show that ∠ B = ∠ C.

Ans. Given: ABC is an isosceles triangle in which AB = AC



To prove: ∠B = ∠C

Construction: Draw AP ⊥ BC

Proof: In ΔABP and ΔACP

$\angle APB = \angle APC = 90^\circ$ [By construction]

$AB = AC$ [Given]

$AP = AP$ [Common]

$\therefore \Delta ABP \cong \Delta ACP$ [RHS congruency]

$\Rightarrow \angle B = \angle C$ [By C.P.C.T.]