SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

1. $\frac{2x}{1+x^2}$

Ans. Let $1 + x^2 = t$

$\therefore 2x\,dx = dt$

$\Rightarrow \int \frac{2x}{1+x^2}dx = \int \frac{1}{t}dt$

$= \log|t| + C$

$= \log|t + x^2| + C$

$= \log(1 + x^2) + C$

2. $\frac{(\log x)^2}{x}$

Ans. Let $\log|x| = t$

$\therefore \frac{1}{x}dx = dt$

$\Rightarrow \int \frac{(\log|x|)^2}{x}dx = \int t^2 dt$

$= \frac{t^3}{3} + C$

$= \frac{(\log|x|)^3}{3} + C$

3. $\frac{1}{x + x\log x}$

Ans. $\frac{1}{x + x\log x} = \frac{1}{x(t + x\log x)}$

Let $1 + \log x = t$

$\therefore \frac{1}{x}dx = dt$

$\Rightarrow \int \frac{1}{x}dx = dt$

$$\Rightarrow \int \frac{1}{x(1+\log x)}dx = \int \frac{1}{t}dt$$

$$= \log|t| + C$$

4. $\sin x \cdot \sin(\cos x)$

Ans. $\sin x \cdot \sin(\cos x)$

Let $\cos x = t$

$$\Rightarrow \int \sin x \cdot \sin(\cos x)dx = -\int \sin t\ dt$$

$$= -[-\cos t] + C$$

$$= \cos t + C$$

$$= \cos(\cos x) + C$$

5. $\sin(ax+b)\cos(ax+b)$

Ans. $\sin(ax+b)\cos(ax+b) = \frac{2\sin(ax+b)\cos(ax+b)}{2} = \frac{\sin 2(ax+b)}{2}$

Let $2(ax+b) = t$

$\therefore\ 2adx = dt$

$$\Rightarrow \int \frac{\sin 2ax+b}{2}dx = \frac{1}{2}\int \frac{\sin t\ dt}{2a}$$

$$= \frac{1}{4a}[-\cos t] + C$$

$$= \frac{-1}{4a}\cos 2(ax+b) + C$$

6. $\sqrt{ax+b}$

Ans. Let $ax+b = t$

$\Rightarrow adx + dt$

$\therefore\ dx = \frac{1}{a}dt$

$$\Rightarrow \int (ax+b)^{\frac{1}{2}}dx = \frac{1}{a}\int t^{\frac{1}{2}}dt$$

$$=\frac{1}{a}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$

$$=\frac{2}{3a}(ax+b)^{\frac{3}{2}}+C$$

7. $x\sqrt{x+2}$

Ans. Let $(x+2)=t$

$\therefore\ dx=dt$

$$\Rightarrow\int x\sqrt{x+2}dx=\int(t-2)\sqrt{t}dt$$

$$=\int x\sqrt{x+2}dx\int(t-2)\sqrt{t}dt$$

$$=\int\left(t^{\frac{3}{2}}-2t^{\frac{1}{2}}\right)dt$$

$$=\int t^{\frac{3}{2}}dt-2\int t^{\frac{1}{2}}dt$$

$$=\frac{t^{\frac{5}{2}}}{\frac{5}{2}}-2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$

$$=\frac{2}{5}t^{\frac{5}{2}}-\frac{4}{3}t^{\frac{3}{2}}+C$$

$$=\frac{2}{3}(x+2)^{\frac{5}{2}}-\frac{4}{3}(x+2)^{\frac{3}{2}}+C$$

8. $x\sqrt{1+2x^2}$

Ans. Let $1+2x^2=t$

$\therefore\ 4xdx=dt$

$$\Rightarrow\int x\sqrt{1+2x2}dx=\int\frac{\sqrt{t}dt}{4}$$

$$=\frac{1}{4}\int t^{\frac{1}{2}}dt$$

$$=\frac{1}{4}\left(\frac{t^{\frac{1}{2}}}{\frac{3}{2}}\right)+C$$

$$=\frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}}+C$$

9. $\left(4x+2\right)\sqrt{x^2+x+1}$

Ans. Let $x^2+x+1=t$

$\therefore\ \left(2x+1\right)dx=dt$

$$\int\left(4x+2\right)\sqrt{x^2+x+1}\ dx$$

$$=\int 2\sqrt{t}\ dt$$

$$=2\int\sqrt{t}\ dt$$

$$=2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$

$$=\frac{4}{3}\left(x^2+x+1\right)^{\frac{3}{2}}+C$$

10. $\frac{1}{x-\sqrt{x}}$

Ans. $\frac{1}{x-\sqrt{x}}=\frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}$

$\text{Let}\left(\sqrt{x}-1\right)=1$

$\therefore\ \frac{1}{2\sqrt{x}}dx=t$

$$\Rightarrow\int\frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}dx=\int\frac{2}{t}\ dt$$

$$=2\log|t|+\ C$$

$$=2\log|\sqrt{x}+t|+\ C$$

11. $\frac{x}{\sqrt{x}}, x > 0$

Ans. Let $x + 4 = t$

$\therefore \; dx = dt$

$$\int \frac{x}{\sqrt{x+4}}dx = \int \frac{(t-4)}{\sqrt{t}}$$

$$= \int\left(\sqrt{t} - \frac{4}{\sqrt{t}}\right)dt$$

$$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 4\left(\frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right) + C$$

$$= \frac{2}{3}(t)^{\frac{3}{2}} - 8(t)^{\frac{1}{2}} + C$$

$$= \frac{2}{3}t - t^{\frac{1}{2}} - 8t^{\frac{1}{2}} + C$$

$$= \frac{2}{3}t^{\frac{1}{2}}(t-12) + C$$

$$= \frac{2}{3}(x+4)^{\frac{1}{2}}(x+4-12) + C$$

$$= \frac{2}{3}\sqrt{x+4}(x-8) + C$$

12. $(x^3 - 1)^{\frac{1}{3}} x^5$

Ans. Let $x^3 - 1 = t$

$\therefore \; 3x^2 dx = dt$

$$\Rightarrow \int (x^3-1)^{\frac{1}{3}} x^5 dx = (x^3-1)^{\frac{1}{3}} x^3 \cdot x^2 dx$$

$$= \int t^{\frac{1}{3}}(t+1)\frac{dt}{3}$$

$$=\frac{1}{3}\left[\frac{t^{\frac{7}{3}}}{\frac{7}{3}}+\frac{t^{\frac{4}{3}}}{\frac{4}{3}}\right]+C$$

$$=\frac{1}{3}\left[\frac{3}{7}t^{\frac{7}{3}}+\frac{3}{4}t^{\frac{4}{3}}\right]+C$$

$$=\frac{1}{7}\left(x^3-1\right)^{\frac{7}{3}}+\frac{1}{4}\left(x^3-1\right)^{\frac{4}{3}}+C$$

13. $\frac{x^2}{\left(2+3x\right)^3}$

Ans. Let $2+3x^3=t$

$\therefore 9x^2dx=dt$

$$\Rightarrow\int\frac{x^2}{\left(2+3x\right)^3}dx=\frac{1}{9}\int\frac{dt}{\left(t\right)^3}$$

$$=\frac{1}{9}\left[\frac{t^{-2}}{-2}\right]+C$$

$$=\frac{-1}{18}\left(\frac{1}{t^2}\right)+C$$

$$=\frac{-1}{18\left(2+3x^3\right)^2}+C$$

14. $\frac{1}{x\left(\log x\right)^m},x>0$

Ans. Let $\log x=t$

$\therefore\ \frac{1}{x}dx=dt$

$$\Rightarrow\int\frac{1}{x\left(\log x\right)^m}dx=\int\frac{dt}{\left(t\right)^m}$$

$$=\left(\frac{t^{-m+1}}{1-m}\right)+C$$

$$=\frac{(\log x)^{1-m}}{(1-m)}+C$$

15. $\frac{x}{9-4x^2}$

Ans. Let $9-4x^2=t$

$\therefore -8x\ dx=dt$

$$\Rightarrow\int\frac{x}{9-4x^2}dx=\frac{-1}{8}\int\frac{1}{t}$$

$$=\frac{-1}{8}\log|t|+C$$

$$=\frac{-1}{8}\log|9-4x^2|+C$$

16. e^{2x+3}

Ans. Let $2x+3=t$

$\therefore\ 2dx=dt$

$$\Rightarrow\int e^{2x+3}dx=\frac{1}{2}\int e^t dt$$

$$=\frac{1}{2}\left(e^t\right)+C$$

$$=\frac{1}{2}e^{(2x+3)}+C$$

17. $\frac{x}{e^{x^2}}$

Ans. Let $x^2=t$

$\therefore\ 2xdt=dt$

$$\Rightarrow\int\frac{x}{e^{x^2}}dx=\frac{1}{2}\int\frac{1}{e^t}dt$$

$$=\frac{1}{2}\int e^{-1}dt$$

$$=\frac{1}{2}\left(\frac{e^{-1}}{-1}\right)+C$$

$$=-\frac{1}{2}e^{x^2}+C$$

$$=\frac{-1}{2e^{x^2}}+C$$

18. $\frac{e^{\tan^{-1}}x}{1+x^2}$

Ans. Let $\tan^{-1}x=1$

$$\therefore \frac{1}{1+x^2}dx=dt$$

$$\Rightarrow\int\frac{e^{\tan^{-1}}}{1+x^2}dx=\int e^{t}dt$$

$$=e^{t}+C$$

$$=e^{\tan-1x}+C$$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

Ans. $\frac{e^{2x}-1}{e^{2x}+1}$

Dividing numerator and denominator by e^x, we obtain

$$\frac{\frac{\left(e^{2x}-1\right)}{e^{x}}}{\frac{\left(e^{2x}+1\right)}{e^{x}}}=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

Let $e^{x}+e^{-x}=t$

$$\therefore \left(e^{x}+e^{-1}\right)dx=dt$$

$$\Rightarrow\int\frac{e^{2x}-1}{e^{2x}+1}dt=\int\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}dt$$

$$=\int \frac{dt}{t}$$

$$=\log|t|+C$$

$$=\log|e^{x}+e^{-x}|+C$$

20. $\dfrac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

Ans. Let $e^{2x}+e^{-2x}=t$

$$\therefore\ \left(2e^{2x}-2e^{-2x}\right)dx=dt$$

$$\Rightarrow \int\left(\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\right)dx=\int\frac{dt}{2t}$$

$$=\frac{1}{2}\int\frac{1}{t}\ dt$$

$$=\frac{1}{2}\log|t|+C$$

$$=\frac{1}{2}\log|e^{2x}+e^{-2x}|+C$$

21. $\tan^{2}\left(2x-3\right)$

Ans. $\tan^{2}\left(2x-3\right)=\sec^{2}\left(2x-3\right)-1$

Let $2x-3=t$

$$\therefore\ 2dx=dt$$

$$\Rightarrow \int\tan^{2}\left(2x-3\right)dx=\int\left[\left(\sec^{2}\left(2x-3\right)\right)-1\right]$$

$$=\frac{1}{2}\int\left(\sec^{2}t\right)dt-\int 1dt$$

$$=\frac{1}{2}\int\sec^{2}t\ dt-\int 1dt$$

$$=\frac{1}{2}\tan t-x+C$$

$$=\frac{1}{2}\tan\left(2x-3\right)-x+C$$

22. $\sec^2(7-4x)$

Ans. Let $7-4x=t$

$\therefore\ -4dx=dt$

$\therefore \int \sec^2(7-4x)dx = \frac{-1}{4}\int \sec^2 t\ dt$

$= \frac{-1}{4}(\tan t)+C$

$= \frac{-1}{4}\tan(7-4x)+C$

23. $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$

Ans. Let $\sin^{-1}x=t$

$\therefore\ \frac{1}{\sqrt{1-x^2}}dx=dt$

$\Rightarrow \int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx = \int t\ dt$

$= \frac{t^2}{2}+C$

24. $\frac{2\cos x-3\sin x}{6\cos x+4\sin x}$

Ans. $\frac{2\cos x-3\sin x}{6\cos x+4\sin x} = \frac{2\cos x-3\sin x}{2(3\cos x+2\sin x)}$

Let $3\cos x+2\sin x=t$

$\therefore (-3\sin x+2\cos x)dx=dt$

$\int \frac{2\cos x-3\sin x}{6\cos x+4\sin x}\ dx = \int \frac{dt}{2t}$

$= \frac{1}{2}\int \frac{1}{t}dt$

$$=\frac{1}{2}\log|t|+C$$

$$=\frac{1}{2}\log\left|2\sin x+3\cos x\right|+C$$

25. $\dfrac{1}{\cos^2 x\left(1-\tan x\right)^2}$

Ans. $\dfrac{1}{\cos^2 x\left(1-\tan x\right)^2}=\dfrac{\cos^2 x}{\left(1-\tan x\right)^2}$

Let $\left(1-\tan x\right)=t$

$\therefore -\sec^2 x dx = td$

$\Rightarrow \int \dfrac{\cos^2 x}{\left(1-\tan x\right)^2}\ dx=\int\dfrac{-dt}{t^2}$

$=-\int t^{-2}dt$

$=+\dfrac{1}{t}+C$

$=\dfrac{1}{\left(1-\tan x\right)}+C$

26. $\dfrac{\cos\sqrt{x}}{\sqrt{x}}$

Ans. Let $\sqrt{2}=t$

$\therefore\ \dfrac{1}{2\sqrt{x}}dx=dt$

$\Rightarrow \int\dfrac{\cos\sqrt{x}}{\sqrt{x}}dt=2\int\cos t\ dt$

$=2\sin t+C$

$=2\sin\sqrt{x}+C$

27. $\sqrt{\sin\ 2x}\cos 2x$

Ans. Let $\sin 2x = t$

$\therefore\ 2\cos 2x dx = dt$

$$\Rightarrow \int \sqrt{\sin 2x} \cos 2x \, dx$$

$$= \frac{1}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{1}{3} t^{\frac{3}{2}} + C$$

$$= \frac{1}{3}(\sin 2x)\frac{3}{2} + C$$

28. $\dfrac{\cos x}{\sqrt{1+\sin x}}$

Ans. Let $1+\sin x = t$

$$\Rightarrow \int \frac{\cos x}{\sqrt{1+\sin x}} dx = \int \frac{dt}{\sqrt{t}}$$

$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= 2\sqrt{t} + C$$

$$= 2\sqrt{1+\sin x} + C$$

29. $\cot x \log \sin x$

Ans. Let $\log \sin x = t$

$$\Rightarrow \frac{1}{\sin x} \cdot \cos x dx = dt$$

$\therefore\ \cot x dx = dt$

$$\Rightarrow \int \cot x \log \sin x \ dt = \int t \, dt$$

$$= \frac{t^2}{2} + C$$

$$=\frac{1}{2}(\log\sin x)^2+C$$

30. $$\frac{\sin x}{1+\cos x}$$

Ans. Let $1+\cos\ x=t$

$$\therefore\ -\sin x\,dx=dt$$

$$\Rightarrow\int\frac{\sin x}{1+\cos x}dx=\int-\frac{dt}{t}$$

$$=-\log|t|+C$$

$$=-|1+\cos x|+C$$

31. $$\frac{\sin x}{(1+\cos x)^2}$$

Ans. Let $1+\cos x=t$

$$\therefore -\sin x\,dx=dt$$

$$\Rightarrow\int\frac{\sin x}{(1+\cos x)^2}dx=\int-\frac{dt}{t^2}$$

$$=-\int t^{-2}dt$$

$$=\frac{1}{t}+C$$

$$=\frac{1}{1+\cos x}+C$$

32. $$\frac{1}{1+\cot x}$$

Ans. Let $I=\int\frac{1}{1+\cot x}dx$

$$=\int\frac{1}{1+\frac{\cot x}{\sin x}}dx$$

$$=\int\frac{\sin x}{\sin x+\cos x}dx$$

$$=\frac{1}{2}\int\frac{2\sin x}{\sin x+\cos x}dx$$

$$=\frac{1}{2}\int\frac{(\sin x+\cos x)+(\sin x-\cos x)}{\sin x+\cos x}dx$$

$$=\frac{1}{2}\int 1\,dx+\frac{1}{2}\int\frac{\sin x-\cos x}{\sin x+\cos x}dx$$

$$=\frac{1}{2}(x)+\frac{1}{2}\int\frac{\sin x-\cos x}{\sin x+\cos x}dx$$

Let $\sin x + \cos x = t \Longrightarrow (\cos x - \sin x)\, d x = dt$

$$\therefore I=\frac{x}{2}+\frac{1}{2}\int\frac{-(dt)}{t}$$

$$=\frac{x}{2}-\frac{1}{2}\log|t|+C$$

$$=\frac{x}{2}-\frac{1}{2}\log|\sin x+\cos x|+C$$

33. $\dfrac{1}{1+\tan x}$

Ans. Let $I=\int\frac{1}{1+\tan x}dx$

$$=\int\frac{1}{1+\frac{\sin x}{\cot x}}dx$$

$$=\int\frac{\cos x}{\cos x+\sin x}dx$$

$$=\frac{1}{2}\int\frac{2\cos x}{\cos x-\sin x}dx$$

$$=\frac{1}{2}\int\frac{(\cos x-\sin x)+(\cos x+\sin x)}{\cos x-\sin x}dx$$

$$=\frac{1}{2}\int 1\,dx+\frac{1}{2}\int\frac{\cos x+\sin x}{\cos x-\sin x}dx$$

$$=\frac{1}{2}(x)+\frac{1}{2}\int\frac{\cos x+\sin x}{\cos x-\sin x}dx$$

Let $\cos x - \sin x = t \Longrightarrow (-\sin x - \cos x)\, d x = dt$

$$\therefore I = \frac{x}{2} + \frac{1}{2}\int \frac{-(dt)}{t}$$

$$= \frac{x}{2} - \frac{1}{2}\log|t| + C$$

$$= \frac{x}{2} - \frac{1}{2}\log|\cos x + \sin x| + C$$

34. $\frac{\sqrt{\tan x}}{\sin x \cos x}$

Ans. Let $I = \int \frac{\sqrt{\tan x}}{\sin x \cos x}\, dx$

$$= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x}\, dx$$

$$= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x}\, dx$$

$$= \int \frac{\sec^2 x\, dx}{\sqrt{\tan x}}$$

Let $\tan x = t \Rightarrow \sec^2 x dx = dt$

$$\therefore l = \int \frac{dt}{\sqrt{t}}$$

$$= 2\sqrt{t} + C$$

$$= 2\sqrt{\tan x} + C$$

35. $\frac{(1+\log x)^2}{x}$

Ans. Let $1 + \log x = t$

$$\therefore \frac{1}{x} dx = dt$$

$$\Rightarrow \int \frac{(1+\log)^2}{x}\, dx = \int t^2 dt$$

$$= \frac{t^3}{3} + C$$

$$=\frac{(1+\log)^3}{3}+C$$

36. $\dfrac{(x+1)(x+\log x)^2}{x}$

Ans. $\dfrac{(x+1)(x+\log x)^2}{x}=\left(\dfrac{x+1}{x}\right)(x+\log x)^2=\left(1+\dfrac{1}{x}\right)(x+\log x)^2$

Let $(x+\log x)=t$

$$\therefore\left(1+\frac{1}{x}\right)dx=dt$$

$$\Rightarrow\int\left(1+\frac{1}{x}\right)(x+\log x)^2\,dx=\int t^2dt$$

$$=\frac{t^3}{3}+C$$

$$=\frac{1}{3}(x+\log x)^3+C$$

37. $\dfrac{x^3\sin\left(\tan^{-1}x^4\right)}{1+x^8}$

Ans. Let $x^4=t$

$$\therefore ax^3\,dx=dt$$

$$\Rightarrow\int\frac{x^3\sin\tan^{-1}x^4}{1+x^8}dx=\frac{1}{4}\int\frac{\sin\tan^{-1}t}{1+t^2}dt$$

Let $\tan^{-1}t=u$

$$\therefore\frac{1}{1+t^2}dt=du$$

From (1), we obtain

$$\int\frac{x^3\sin\left(\tan^{-1}x^4\right)dx}{1+x^8}=\frac{1}{4}\int\sin u\,du$$

$$=\frac{1}{4}(-\cos u)+C$$

$$=\frac{-1}{4}\cos\left(\tan^{-1}t\right)+C$$

$$= \frac{-1}{4}\cos\left(\tan^{-1} x^4\right) + C$$

38. $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$ equals

(A) $10^x - x^{10} + C$ (B) $10^x + x^{10} + C$

(C) $\left(10^x - x^{10}\right)^{-1} + C$ (D) $\log\left(10^x + x^{10}\right) + C$

Ans. Let $x^{10} + 10^x = t$

$$\therefore \left(10x^9 + 10^x \log_e 10\right)dx = dt$$

$$\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$$

$$= \log t + C$$

$$= \log\left(10^x + x^{10}\right) + C$$

39. $\int \frac{dx}{\sin^2 x \cos^2 x}$ equals

A. $\tan x + \cot x + C$ B. $\tan x - \cot x + C$ C. $\tan x \cos x + C$ D. $\tan x - \cot 2x + C$

Ans. Let $I = \int \frac{dt}{\sin^2 x \cos^2 x} dx$

$$= \int \frac{dt}{\sin^2 x \cos^2 x}$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2$$

$$= \tan x - \cot x + C$$

Hence, the correct Answer is B.