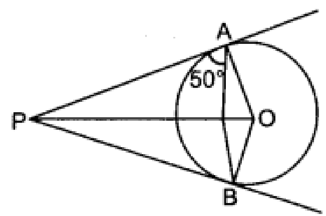
Question 1.

From an external point P, tangents PA and PB are drawn to a circle with centre O. If ∠PAB = 50°, then find ∠AOB.

Solution:



Given, $\angle PAB = 50^\circ$

$\angle PAB + \angle OAB = 90^\circ$

[$\because$ angle between radius OA and tangent PA is 90°]

$\Rightarrow 50^\circ + \angle OAB = 90^\circ$

$\Rightarrow \angle OAB = 90^\circ - 50^\circ = 40^\circ$

Now, $PA = PB$ [$\therefore$ tangents from an external point are same]

$\Rightarrow \angle PBA = \angle PAB$

$\Rightarrow \angle PBA = 50^\circ$

$\angle PBA + \angle OBA = 90^\circ$ [$\therefore$ angle between radius OB and tangent PB is 180°]

$\Rightarrow 50^\circ + \angle OBA = 90^\circ$

$\Rightarrow \angle OBA = 90^\circ - 50^\circ = 40^\circ$

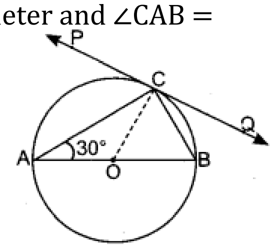
Now in ΔAOB we have

$\angle AOB + \angle ABO + \angle BAO = 180^\circ$ [$\because$ sum of angles in triangle is 180°]

$\Rightarrow \angle AOB + 40^\circ + 40^\circ = 180^\circ \Rightarrow \angle AOB = 180^\circ - 80^\circ = 100^\circ$

Question 2.

In given figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and ∠CAB = 30°, find ∠PCA



Solution:

Construction: Join AO.

Given: PQ is tangent. AB is diameter ∠CAB = 30°.

To Find: ∠PCA

Solution: In ΔAOC, $AO = CO$ ($\because$ Equal radii)

$\angle CAO = \angle OCA$ ($\because$ Angles opposite to equal sides are equal)

or $\angle CAB = \angle OCA$

But, $\angle CAB = 30^\circ$ So, $\angle OCA = 30^\circ$ (*i*)

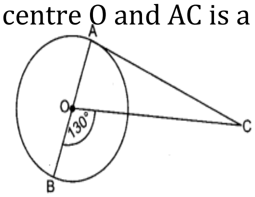
Since, $OC \perp PQ$ ($\because$ Tangent is perpendicular to radius at point of contact)

$\Rightarrow \angle PCO = 90^\circ \Rightarrow \angle OCA + \angle PCA = 90^\circ \Rightarrow 30^\circ + \angle PCA = 90^\circ$

$\therefore \angle PCA = 60^\circ$

Question 3.

In figure given, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130°$, then find $\angle ACO$.



Solution:

$$\angle AOC + \angle BOC = 180°$$

[$\because$ Linear Pair Axiom]

$$\angle AOC + 130° = 180°$$

$$\angle AOC = 180° - 130°$$

$$\angle AOC = 50°$$

Now, $\angle OAC = 90°$ [angle between radius OA and tangent AC is 90°]

Now, in ΔAOC,

$$\angle OAC + \angle AOC + \angle ACO = 180°$$ [$\because$ sum of angles in triangle is 180°]

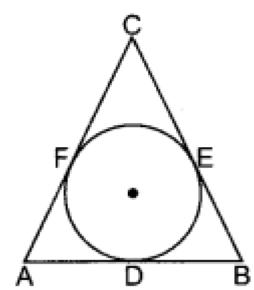
$$90° + 50° + \angle ACO = 180°$$

$$\angle ACO = 180° - 140°$$

$$\angle ACO = 40°$$

Question 4.

In given figure, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF



Solution:

Given, AB = 12 cm, CA = 10 cm, BC = 8 cm

Let $AD = AF = x$ [$\because$ Tangent drawn from external point to circle are equal]

$\therefore$ $DB = BE = 12 - x$ and $CF = CE = 10 - x$

$BC = BE + EC \Rightarrow 8 = 12 - x + 10 - x$

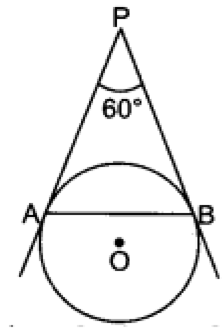
$\Rightarrow$ $x = 7$

$\therefore$ AD = 7 cm, BE = 5 cm and CF = 3 cm

Question 5.

If given figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and $\angle APB = 60°$. Find the length of chord AB.

Solution:



In ΔAPB we have $AP = BP$

$\Rightarrow \angle PAB = \angle PBA$

[$\because$ Tangents from an external point are equally inclined to segment joining centre to point]

Let $\angle PAB = x$,

then in ΔAPB, $x + x + 60° = 180°$

$2x = 180° - 60° = 120°$

$x = 60°$

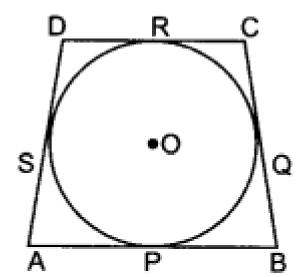
As all three angles of ΔAPB are 60°. So ΔAPB is an equilateral triangle.

Hence $AP = BP = AB = 5$ cm

Question 6.

In figure, a quadrilateral ABCD is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that AB + CD = BC + DA.

Solution:



We know that tangents drawn to a circle from an outer points are equal.

So, $AP = AS, BP = BQ,$

$CR = CQ$ and $DR = DS$.

Now, consider

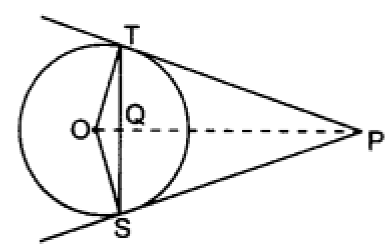
$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$\Rightarrow AB + CD = AD + BC$$

Hence proved.

Question 7.

In given figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r.If PO = 2r, show that ∠OTS =∠OST = 30°.



Solution:

Let $\angle TOP = \theta$

In right triangle OTP we have

$\therefore \quad \cos\theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2} = \cos 60° \Rightarrow \theta = 60°$

Hence $\angle TOS = 2 \times 60 = 120°$ [$\because \angle TOP = \angle POS$ as angles opposite to equal tangent are equal]

In ΔOTS, we have $OT = OS$ [$\because$ Equal radii]

$\Rightarrow \quad \angle OTS = \angle OST$ [$\because$ Angle opposite to equal sides are equal]

In ΔOTS,

$$\angle OTS + \angle OST + \angle TOS = 180°$$

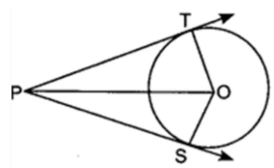
$$2\angle OST = 60°$$

$\therefore \quad \angle OST = \angle OTS = 30°$

Hence proved.

Question 8.

In given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that $\angle SPT = 120°$, Prove that OP = 2PS



Solution:

Let $PT = x = PS$ [$\because$ Tangent drawn from external point to circle are equal]

$\angle SPT = 120°$

In ΔOTP and ΔOSP, $\angle OTP = \angle OSP$

[$\because$ each equal to 90°, since tangent perpendicular r radius]

$OT = OS$ [$\because$ Equal radii]

$OP = OP$ [common]

$\Rightarrow \quad \Delta OSP \cong \Delta OTP$ [$\because$ By SAS congruence rule]

$\therefore \quad \angle TPO = \angle SPO$ [$\because$ By CPCT]

$\Rightarrow \quad \angle TPO = \frac{1}{2}\angle SPT = \frac{1}{2} \times 120 = 60°$

In ΔOTP, $\frac{OP}{x} = \text{Sec } 60°$

$\Rightarrow \quad \frac{OP}{x} = 2 \Rightarrow OP = 2x \Rightarrow OP = 2PS$

Hence proved.

Question 9.

In given figure, there are two concentric circles of radii 6 cm and 4 cm with centre O. If AP is a tangent to the larger circle and BP to the smaller circle and length of AP is 8cm, find the length of BP

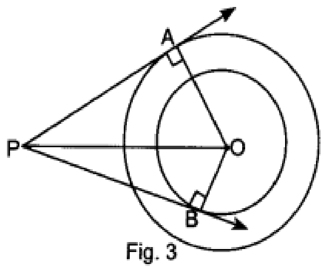


Fig. 3

Solution:

$$OA = 6\text{ cm } [\because \text{ Given radius}]$$
$$OB = 4\text{ cm } [\because \text{ Given radius}]$$
$$AP = 8\text{ cm}$$

In ΔOAP,

$$OP^2 = OA^2 + AP^2 = 36 + 64 = 100 \, [\because \text{ Pythagoras theorem}]$$

$\Rightarrow$

$$OP = 10\text{ cm}$$

In ΔOBP,

$$BP^2 = OP^2 - OB^2 = 100 - 16 = 84 \, [\because \text{ Pythagoras theorem}]$$
$$BP = 2\sqrt{21}\text{ cm}$$

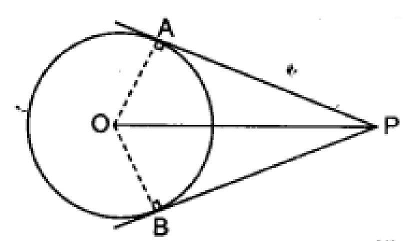
Long Answer Type Questions [4 Marks]

Question 10.

Prove that the lengths of tangents drawn from an external point to a circle are equal

Solution:

Given: A circle $C(O, r)$, P is a point outside the circle and PA and PB are tangents to a circle.



To Prove: PA = PB

Construction: Draw OA, OB and OP.

Proof: Consider triangles OAP and OBP.

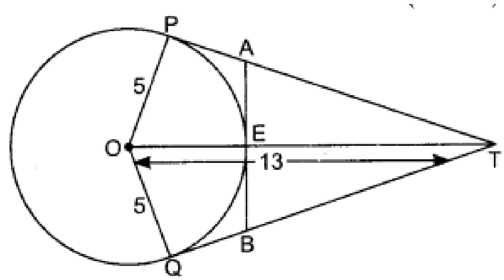
$$\angle OAP = \angle OBP = 90^\circ \quad ...(i)$$

[Radius is perpendicular to the tangent at the point of contact]

	OA	= OB (radii)	...(*ii*)
OP is common			...(*iii*)
∴	ΔOAP	$\cong \Delta OBP$ (RHS)	[from (*i*), (*ii*) and (*iii*)]
Hence,	AP	= BP	(CPCT)

Question 11.

In given figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



Solution:

In ΔOPT, $OP^2 + PT^2 = OT^2$ [∵ Pythagoras theorem]

$PT = \sqrt{OT^2 - OP^2}$

$= \sqrt{169 - 25} = 12$ cm

and $TE = OT - OE = 13 - 5 = 8$ cm

Let $PA = AE = x$ [tangent from outer point A]

In ΔTEA, $TE^2 + EA^2 = TA^2$ [∵ Pythagoras theorem]

$(8)^2 + (x)^2 = (12 - x)^2$

$64 + x^2 = (12 - x)^2$

⇒ $64 + x^2 = 144 + x^2 - 24x$

⇒ $80 = 24x \Rightarrow x = 3.3$ cm

Thus $AB = 2 \times 3.3$ cm $= 6.6$ cm [∵ AE = EB, as AB is tangent to circle at E]

Question 12.

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

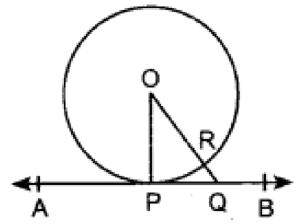
Solution:

Given: A circle $C(O, r)$ and a tangent AB at a point P.

To prove: $OP \perp AB$

Construction: Take any point Q other than P on the tangent AB.

Join OQ, intersecting circle at R.



Proof: We have, $OP = OR$ [Radii]

$OQ = OR + RQ$

$\therefore \quad OQ > OR \Rightarrow OQ > OP$ [$\because$ OR = OP = radius]

Thus, $OP < OQ$, i.e. OP is shorter than any other segment joining O to any point of AB.

But among all line segments, joining point O to point on AB, shortest one is perpendicular from O on AB.

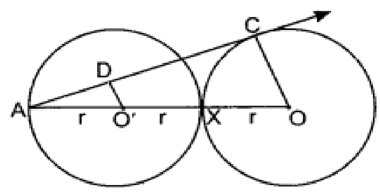
Hence, $OP \perp AB$

Question 13.

In given figure, two equal circles, with centers O and O', touch each other at X. OO' produced meets the circle with Centre O' at A.AC is tangent to the circle with centre O, at the point C. O'D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$.

Solution:

AC is tangent to the circle with centre O.



In $\Delta ADO'$ and ΔACO, $\angle ADO' = \angle ACO$ (each 90°)

$\angle DAO = \angle CAO$ (common)

$\therefore$ By AA criterion, $\frac{AO'}{AO} = \frac{DO'}{CO}$ [$\because$ corresponding parts of similar triangle]

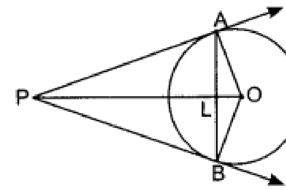
$AO = AO' + O'X + XO = r + r + r = 3r$

$\frac{DO'}{CO} = \frac{r}{3r}$ [$\therefore$ AO = AO' + O'X + XO = 3AO]

$\Rightarrow \quad \frac{DO'}{CO} = \frac{1}{3}$

Question 14.

In given figure, AB is a chord of a circle, with centre O, such that AB = 16 cm and radius of circle is 10 cm. Tangents at A and B intersect each other at P. Find the length of PA



Solution:

Let $PL = x$

As OP is perpendicular bisector of AB. Then

$$AL = BL = 8 \text{ cm}$$

In ΔALO, $OL^2 = OA^2 - AL^2 = 10^2 - 8^2 = 36 \Rightarrow OL = 6 \text{ cm}$

$AP^2 = OP^2 - OA^2$ [$\because$ Pythagoras theorem]

In ΔOAP, $AP^2 = (x+6)^2 - 10^2$

$AP^2 = AL^2 + PL^2$ [$\because$ Pythagoras theorem]

In ΔALP, $AP^2 = x^2 + 64$

Now, $(x+6)^2 - 10^2 = x^2 + 64$

$$x^2 + 12x + 36 - 100 = x^2 + 64$$

$$\Rightarrow \quad 12x = 128$$

$$\Rightarrow \quad x = \frac{128}{12}$$

$$= \frac{32}{3} \text{ cm}$$

From ΔALP, $AP^2 = \left(\frac{32}{3}\right)^2 + 64$

$$= \frac{1024}{9} + 64$$

$$= \frac{1024 + 576}{9} \text{ cm}$$

$$AP^2 = \frac{1600}{9} \text{ cm}$$

$$AP = \frac{40}{3} \text{ cm} = 13.3 \text{ cm}$$